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Non-Homogeneous Cubic Equation with Three Unknowns

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Abstract

The non-homogeneous cubic equation with three unknowns represented by $3(x^2+y^2)-5xy+2(x+y)+4=27z^3$ is analyzed for finding its non-zero distinct integral solutions. Three different methods have been presented for determining the integral solutions of the non-homogeneous cubic equation under consideration. Employing the integral solutions of the above equation, a few interesting relations between special numbers are exhibited.

Keywords: Ternary,cubic,Non-homogeneous, Integral solutions.

MSc 2000 Mathematics Subject Classification: 11D25

Introduction

Integral solutions for the cubic homogeneous or non-homogeneous Diophantine equations is an interesting concept, as it can be seen from [1,2,3].In [4-24],a few special cases of ternary cubic Diophantine equations are analyzed. In this communication, we present the integral solutions of yet another cubic equation $3(x^2+y^2)-5xy+2(x+y)+4=27z^3$ along with a few interesting properties

Notations:

$$t_{m,n}=n\left(1+\frac{(n-1)(m-2)}{2}\right)$$.

$$P_n^m=\left(\frac{n(n+1)}{6}\right)[(m-2)n+(5-m)]$$

$$SO_n=n(2n^2-1)$$

$$S_n=6n(n-1)+1$$

$$\Pr_n=n(n+1)$$

Method of analysis

Consider the non-homogeneous ternary cubic Diophantine equation

$$3(x^2+y^2)-5xy+2(x+y)+4=27z^3 \tag{1}$$

Introducing the linear transformations

$$x=u+v,\ y=u-v \tag{2}$$

in (1),it is written as

$$(u+2)^2+11v^2=27z^3 \tag{3}$$

We employ different ways of solving (3) and thus, different patterns of integer solutions to (1) are illustrated below.

Pattern:I

Write 27 as

$$27=(4+i\sqrt{11})(4-i\sqrt{11}) \tag{4}$$

Assume

$$z=a^2+11b^2 \tag{5}$$

where a and b are non-zero distinct integers.

Using (4) and (5) in (3) and employing the method of factorization, define

$$(u+2)+i\sqrt{11}v=(4+i\sqrt{11})(a+i\sqrt{11}b)^3$$

Equating real and imaginary parts in the above equation, we get,

$$u+2=4a^3-132ab^2-33a^2b+121b^3$$

$$v=a^3-33ab^2+12a^2b-44b^3$$

Substituting the values of u, v in (2),we have

$$x(a,b)=5a^3-165ab^2-21a^2b+77b^3-2$$

$$y(a,b)=3a^3-99ab^2-45a^2b+165b^3-2 \tag{6}$$

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Thus (5) and (6) represent the non-zero distinct integer solutions of (1).

Properties:

- $x(a,1)+y(a,1)-16P_a^5+12S_a+2t_{4,a}\equiv 0(\bmod 2)$
- $x(1,b)+y(1,b)+z(1,b)-121So_b+253P_{rb}\equiv 5(\bmod 308)$
- $x(a,a+1)+z(a,a+1)+208P_a^5+S_a-4t_{3,a}\equiv 7(\bmod 10)$
- $y(a,1)+x(a,1)+66P_{ra}+396t_{3,a}-198t_{4,a}-238$ is a cubical integer.
- $3x(1,b)-5y(1,b)+162So_b+540P_b^5-270t_{4,b}=4$
- $y(1,b)-z(1,b)-82So_b-2P_b^5-37P_{rb}+154t_{4,b}$ is a nasty number.
- $z(1,2^n)-10Ky_n+89$ is a perfect square.

PATTERN: II

Instead of (4),write 27 as

$$27=\frac{(3+i3\sqrt{11})(3-i3\sqrt{11})}{4} \qquad (7)$$

Substituting (6) in (3) and proceeding as in Pattern I, the non-zero distinct integral solutions to (1) are given by

$$x(a,b)=3a^3-99ab^2-45a^2b+165b^3-2$$
$$y(a,b)=-2a^3+18ab^2-12a^2b+12b^3$$
$$z(a,b)=a^2+11b^2$$

Properties:

- $x(a,1)+y(a,1)-6P_a^5+198t_{3,a}+3t_{4,a}=359$
- $x(1,b)-y(1,b)+66P_b^5+75t_{4,b}-9P_{rb}=3$
- $x(a,1)+y(a,1)+z(a,1)-So_a+196t_{3,a}-370$ is a cubical integer.
- $108z(a,1)-2y(a,1)-796$ is a nasty number**.**

PATTERN: III

Write the equation (3) as

$$(u+2)^2+11v^2=27*z^3*1 \qquad (8)$$

1 can be written as

$$1=\frac{[(2n^2-2n-5)+i\sqrt{11}(2n-1)][(2n^2-2n-5)+i\sqrt{11}(2n-1)]}{(2n^2-2n+6)^2} \qquad (9)$$

Using (4),(5) and (9) in (8) and proceeding as above, the non-zero distinct integer solutions to (1) are obtained as

$$x(a,b)=(2n^2-2n+6)^2[(2n^2-2n-5)(5a^3-165ab^2-21a^2b+77b^3)-(2n-1)(7a^3-231ab^2+165a^2b-605b^3)]-2$$
$$y(a,b)=(2n^2-2n+6)^2[(2n^2-2n-5)(3a^3-99ab^2-45a^2b+165b^3)-(2n-1)(15a^3-495ab^2+99a^2b-363b^3)]-2$$
$$z(a,b)=(2n^2-2n+6)^2[a^2+11b^2]$$

For simplicity and clear understanding, we present the solutions with the properties when (i) n=1 and (ii) n=2.

For **n=1,**the corresponding integer solutions to (1) are

$$x(a,b)=144[-8a^3+264ab^2-15a^2b+55b^3]-2$$
$$y(a,b)=216[-5a^3+165ab^2+21a^2b-77b^3]-2$$
$$z(a,b)=36[a^2+11b^2]$$

Properties:

- $x(a,1)+y(a,1)+4464P_a^5-9216$ is divisible by 2.
- $15x(1,b)-16y(1,b)-104976So_b-349920P_b^5+174960t_{4,b}=2$
- $15x(a,1)-16y(a,1)+2916z(a,1)=1539650$

For **n=2**,the integer solutions to (1) are

$$x(a,b)=200[-13a^3+429ab^2-237a^2b+869b^3]-2$$
$$y(a,b)=400[-12a^3+396ab^2-63a^2b+231b^3]-2$$
$$z(a,b)=100[a^2+11b^2]$$

Properties:

- $x(a,1)-y(a,1)-4400P_a^5-200t_{730,a}+97200t_{4,a}=81400$
- $x(1,b)-y(1,b)+66z(1,b)-1628000P_b^5+81400t_{4,b}\equiv 0(\bmod 5)$
- $x(1,b)+y(1,b)-532400P_b^5+44000t_{3,b}\equiv 0(\bmod 2)$

Conclusion

It is worth to note that using (6) and (8) in (7) and proceeding as in pattern. III, we get a different set of non-zero distinct integral solutions to (1).As the cubic equations are rich in variety,one may search for integer solutions to other choices of ternary cubic equations as well as cubic equation with multivariables(>3).

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